The Verbal Math Lesson - Fractions

Sample lessons - First three lessons contain the answers. These are for the teacher. The second half contains questions but no answers. These are for the student.

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Lessons

Introduction

One day while browsing in an antiquarian book store in San Francisco, we noticed a tattered old book called Verbal Math. The idea of the book was to teach math verbally, without writing the problems down. Intrigued, we bought the book and tried it with our son who disliked worksheets. He loved it. Suddenly math became more like a game for him than work.

We learned that mental math used to be a very popular method of teaching children math. It was free of the tediousness of writing while teaching children efficient methods of solving math problems. Wanting to reintroduce this method in a step-by-step fashion, we have created a series of books using this approach. We call it verbal math. This simple idea will free your student from the drudgery of handwriting and will turn math into a mental game.

In this book, we bring this time-honored approach to help your student become proficient with fractions. After the four basic arithmetic operations, fractions are the most fundamental of math concepts. All progress in math will depend on how well the student understands and is able to work with fractions. If your student is not proficient with fractions, it is likely that he or she will begin to dislike math in higher grade levels. The fractions are the first significant roadblock in math education and if not mastered, will have negative future impact.

With Verbal Fractions, your student will learn, in a gradual manner, what fractions mean. Follow the lessons and see your student do even complex - looking problems quickly and accurately. All without pencil or paper. When time comes to learn algebra, your student will be miles ahead of his or her peers.

Here is an example of the type of problem your student will be able to do quickly by the end of this course, without any figuring on paper.

The distance from Mount Joy to Harrisburg is 25 miles, and (4/5) of this distance is (5/8) of (4/9) of the distance from Harrisburg to Minton. What is the distance to Minton?

This course introduces a special way of analyzing math problems. The method is based on understanding of unit values. The first couple of problems in each lesson show the process.

Please instruct your student to follow this method even though he or she may know of other strategies. Using this method will make more complicated problems easier to solve. The purpose of this course is to help student develop a conceptual understanding of fractions.

SOME GUIDELINES

1. You must start this course at lesson one, even if the problems seem very simple to your child.

2. This course requires a teacher or parent to work with the student. Problems must be read to the student. You may need to guide the student, helping as needed with difficult problems, and using the solutions and methods given in the book. The answers are there just for you. (If you feel it necessary to give your student a copy of the problems, this book-without the answers-is available for purchase in eBook form at our website or at Amazon.com.)

3. All of these problems can be done mentally. When you read the problem to the child, he or she may write

down the numbers but all calculations should be done mentally.

4. Follow the method that is given at the beginning of each lesson. The solutions given should be explained to the student if she has difficulty.

5. Most students can easily do half a lesson a day. Some longer lessons can be spread over a week. The problems do get more difficult towards the end of the book. The student will not be able to do the problems in the later chapters until he or she has done the preceding ones.

6. Subordinate speed to accuracy, but do not neglect speed. The ability to answer these problems quickly results in mastery of the concepts.

7. The course is suitable for students in 4th-6th grades and above depending on the child's ability with arithmetic.

8. Older students who are not confident with fractions or weak in algebra will benefit from this course as well. College entrance exams often have problems of this type on the tests. This book is an excellent refresher course for boosting math scores in standardized tests.

9. If your student has difficulty with understanding or retaining these problem by just listening to the numbers, please consider buying the Verbal Fractions Student Copy, without the answers. The student can then have his own copy but of course without the answers or the solutions. It is available both in PDF and eBook formats.

10. After you have done approximately two-thirds of this book, please consider starting the companion book on Percents.

11. Last few chapters are indeed difficult for many students and are not necessary for proficiency.

Please let us know how this book works for you.

With best wishes,

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Displaying fractions in an eBook, as they are in a type-set book, is difficult because of current format limitations. In this eBook, all fractions are enclosed in parenthesis. For example, (2/3) should be read as two-thirds and 3(5/6) as three-and-five-sixths.

Find any typos, errors in this book? Let us know. We appreciate your understanding in this matter and will give you a free coupon for an eBook in this series.

Lesson one-Halves, Thirds, and Fourths

IF I divide an apple into two equal parts, what is one of these parts called? What are two of these parts called? ***Ans:*** One-half; a whole or one.

1. How many halves are in one apple? ***Ans:*** 2.
2. What is one-half of 4? of 8? of 10? of 12? ***Ans:*** 2; 4; 5; 6.
3. What is one-half of 14? of 16? of 18? of 20? ***Ans:*** 7; 8; 9; 10.
4. What is one-half of 22? of 26? of 28? of 32? ***Ans:*** 11; 13; 14; 16.
5. If one pound of sugar costs 100 cents, what will one-half pound of sugar cost? ***Ans:*** 50 cents.
6. If a pound of coffee costs $10, then how many half pounds can you buy for $10? ***Ans:*** Two half-pounds.
7. If a pound of sugar costs $5, then how many half-pound boxes can you buy for $5? ***Ans:*** Two half-pound boxes.
8. Paul picked 18 apples and gave one-half to his brother. How many apples did he give to his brother? ***Ans:*** 9 apples.
9. Thompson bought 14 books, and sold one-half of them when the school term was over. How many of his books did he sell? ***Ans:*** He sold one-half of 14, which is 7 books.
10. Phoebe had 40 peaches and gave one-half of them away. How many does she have left? ***Ans:*** 20.
11. If I divide an apple into 3 equal parts, what is one of these parts called? ***Ans:*** Each part is called one-third.
12. How many thirds are in one apple? ***Ans:*** 3.
13. What are 2 parts and 3 parts of an apple cut into three pieces called individually? ***Ans:*** Two parts are called 2 thirds and 3 parts is called a whole.
14. How many one-thirds do you need to make a whole? ***Ans:*** Three one-thirds make a whole.
15. What is one-third of 6? of 9? of 12? of 15? ***Ans:*** 2; 3; 4; 5.
16. What is one-third of 21? of 24? of 30? of 36? ***Ans:*** 7; 8; 10; 12.
17. ames had $30 and spent one-third of it. How much did he spend? ***Ans:*** One-third of $30 is $10. He spent $10.
18. Sanjay had 9 pears and Thomas had one-third as many as Sanjay. How many pears did Thomas have? ***Ans:*** Thomas had one-third of 9, which is 3 pears.
19. Lucy had 21 stamps and gave Mary one-third of them. How many stamps did she give to Mary? ***Ans:*** One-third of 21 is 7. Lucy gave Mary one-third, or 7 stamps.

20. The shop had 42 roses, and sold one-third of them to Bob. How many does he have left? ***Ans:*** One-third of 42 is 14. So 2 thirds is twice that many, or 28. After selling one-third, the shop has 2 thirds left, which is 28 roses.

21. What is 2 thirds of 9? ***Ans:*** One-third of 9 is 3. If one-third of 9 is 3, then 2 thirds of 9 is 2 times 3, which is 6.

Note—In doing these problems, we will first find the unit fraction value and then multiply it with the numerator to get the answer. The unit fraction in problem 21 is (1/3). Its value is 3. In all subsequent problems, always have student compute the unit fraction first. This is the only way to develop speed and understanding.

22. What is 2 thirds of 6? of 12? of 15? of 18? ***Ans:*** (3 x 2) = 4; (4 x 2) = 8; (5 x 2) = 10; (6 x 2) = 12. First find the unit fraction. One-third of 6 is 2. Two-thirds is twice that many, or 4.

Note—Ask student to first find the unit fraction then multiply it by 2.

23. What is 2 thirds of 24? of 30? of 27? of 33? ***Ans:*** 16; 20; 18; 22.

    One-third of 24 is 8. Two-thirds is twice that many, or 16.

    One-third of 30 is 10. Two-thirds is twice that many, or 20.

    One-third of 27 is 9. Two-thirds is twice that many, or 18.

    One-third of 33 is 11. Two-thirds is twice that many, or 22.

24. What is one-third of 123? What is two-thirds of 123? ***Ans:*** 41; 82.

25. What is one-third of 222? What is two-thirds of 222? ***Ans:*** 74; 148.

26. John had $21, and gave 2 thirds of his money to Sarah. How much did he give to Sarah? ***Ans:*** One-third of 21 is 7. Two-thirds is twice that many or 14. So he gave Sarah $14.

27. There were 27 cherries in the bowl. You ate 2 thirds of them. How many did you eat? ***Ans:*** One-third of 27 is 9. Two-thirds is twice that many, or 18. I ate 18 cherries.

28. If you give away one-third of something, how many thirds do you have left? ***Ans:*** 2 thirds.

29. 29. Rudy had a rock collection of 33 rocks. He gave his sister for her birthday 2 thirds of it. How many did he keep? ***Ans:*** He has one-third left which is 11. Rudy kept 11 rocks.

30. Daniel had $36 and lost 2 thirds of it somewhere. How much money does he have left? ***Ans:*** After losing 2 thirds, Daniel has one-third left. One-third of 36 is 12.

31. If I divide an apple into 4 equal parts, what are 1, 2, and 3, of these parts called? ***Ans:*** One = one-fourth, 2 = two-fourths or one-half, 3 = three-fourths.

32. How many fourths of an apple are in a whole apple? ***Ans:*** 4 fourths make a whole apple.

33. What is one-fourth of 4? of 8? of 20? of 32? ***Ans:*** 1; 2; 5; 8.

34. What is one-fourth of 12? of 24? of 16? of 48? ***Ans:*** 3; 6; 4; 12.

35. What is 2 fourths of 24? of 16?; of 28? of 36? ***Ans:*** 12; 8; 14; 18.

Note—Ask student to first find the unit fraction, such as what (1/4) of 24.

36. If one-fourth of 20 is 5, then what is 2 fourths? ***Ans:*** It is 2 times 5 or, 10.

37. What is 3 fourths of 20? of 24? of 12? of 16?

Note—First compute one-fourth and then multiplied it by 3. This is the easiest way of doing fractions. Try to make it a habit. It makes fractions simple.

38. What is 3 fourths of 120? ***Ans:*** One-fourth of 120 is 30. Three-fourths is 3 times that or 90.

39. The ski shop had 44 snow boards. The shop rented 2 fourths of them on Friday. How many snow boards are left in the shop? ***Ans:*** One-fourth of 44 is 11. Two-fourths is twice that much, or 22. The shop has 22 snow boards left.

40. If a meter of cloth costs $8, then what will 3 fourths of a meter cost? ***Ans:*** One-fourth of 8 is 2. Three-fourths is 3 times as much or 6. The cloth will cost $6.

41. If I give away one-third of some thing, how many thirds do I have left? ***Ans:*** two-thirds.

42. What is one-half and one-third of 24? ***Ans:*** 12 and 8.

43. Jessica had 24 gum-balls. She gave half of them to Margie and a third of them to Cathy. How many gum-balls does she have left? ***Ans:*** One-half of 24 is 12. One-third of 24 is 8. She gave 12 gum-balls to Margie and 8 balls to Cathy, or a total of 20 gum-balls. She had 24, so she has 24 - 20 = 4 left.

44. Harley is 24 years old and Townsend is 8 fourths as old. How old is Townsend? ***Ans:*** One-fourth of 24 is 6. Eight-fourths is 8 times as much, or $6 \times 8 = 48$. Townsend is 48 years old.

45. What is the sum of one half and one-third of 36? ***Ans:*** 18 plus 12, 30.

46. A farmer harvested 36 melons. A shop bought half of them, and one-third were too green to be sold. How many ripe melons does the farmer have left? ***Ans:*** The number of melons sold plus which is 18 plus those he can not sell, which are 12 adds to 30. So he has 6 ripe melons left.

47. A store owner has 40 boxes of sneakers. He sold 3 fourths of them. How many boxes does he have left? He then bought one-third as much as he sold. How many boxes of sneakers does the store have now? ***Ans:*** One-fourth of 40 is 10. The owner sold 3 fourths of 40, which is 3 times 10, or 30. Then he buys one-third of 30, which is 10. He had 40, sold 30 and bought 10 boxes. So the total number of boxes he has now is 40 - 30 + 10 = 20 boxes.

48. A store owner has 20 jackets. She sold 2 fourths of them. How many does she have left? She then bought one-half as much as she sold. How many are there now? ***Ans:*** 15.

49. What is an another way of saying one-fourth? ***Ans:*** One quarter, or just a quarter.

50. Why is the coin “quarter” called a quarter?

Ans: The coin called quarter is worth 25 cents. There are 100 cents in a dollar and 25 cents is one-fourth of that. That is why it is called a quarter, or a quarter-dollar.

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Lesson 2 - Fifth, Sixth, and Seventh

IF you divide an orange into 5 equal parts, what are 1, 2, 3, and 4 of these parts called?

Ans: One-fifth, 2 fifths, 3 fifths. 4 fifths.

1. How many fifths are in one orange? ***Ans:*** 5.
2. What is one-fifth of 10? of 25? of 15? of 30? ***Ans:*** 2; 5; 3; 6.
3. What is 2 fifths of 15? of 30? of 45? of 20? ***Ans:*** 6; 12; 18; 8.
   Note—First compute one-fifth and then multiply it by 2.
4. What is 3 fifths of 10? of 30? of 25? of 55? ***Ans:*** 6; 18; 15; 33.
   Note—First compute one-fifth and then multiply it by 3.
5. What is 4 fifths of 55? of 35? of 40? of 50? ***Ans:*** 44; 28; 32; 40.
   Note—First compute one-fifth and then multiply it by 4.
6. Mary has 15 blue shells, and Rachel has 2 fifths as many. How many blue beads does Rachel have? ***Ans:*** One-fifth of 15 is 3, so Rachel has 6 blue shells.
7. Susan is 25 years old, and her sister is 4 fifths as old. How old is her sister? ***Ans:*** 20 years.
8. Roland is 35 years old, and his sister is 4 fifths as old. How old is his sister? ***Ans:*** 28 years.
9. A horse cost $1000, and a saddle cost 3 fifths as much as the horse. What is the cost of the saddle?

   Ans: One-fifth of $1000 is $200. Three-fifths is three times as much or $600. The cost of saddle is $600.
10. Mary had 40 lambs and she lost 20 of them. After a lot of searching, she found 3 fifths of her lambs. How many lambs does she have now?
    Ans: If Mary lost 20 lambs, she had left 40 - 20, or 20 lambs. One-fifth of 20 is 4, so 3 fifths of 20 is 12. So she has 20, and the 12 she found or, 12 + 20 = 32, the number of lambs Mary has left.
11. A dairyman owned 50 cows. He sold 4 fifths of them, and then bought 32 new cows. How many does he have now?
    Ans: (4/5) of 50 is 40. He has 10 left and he bought 32 more, so he has 42 cows now.
12. If you cut a melon into 6 equal parts, what are these parts called? ***Ans:*** One-sixth, 2 sixths, 3 sixths, 4 sixths, 5 sixths.
13. How many sixths are there in a single thing? ***Ans:*** 6.
14. What is 2 sixths of 24? of 18? of 36? of 60? ***Ans:*** 8; 6; 12; 20.
    Note—First compute one-sixth of the number and then multiply it by 2.
15. What is 3 sixths of 12? of 42? of 30? of 66? ***Ans:*** 6; 21; 15; 33.

16. What is 4 sixths of 6? of 36? of 48? of 54? ***Ans:*** 4; 24; 32; 36.

17. What is 5 sixths of 18? of 54? of 24? of 72? ***Ans:*** 15; 45; 20; 60.

18. Rob went to a festival and brought back 48 flags. He gave 3 sixths of the flags to Megan and 2 sixths to Morgan. How many flags did he give to both? ***Ans:*** 24 flags to Megan, 16 to Morgan.

19. What will 5 sixths of 36 meters of cloth cost at the rate of $2 a meter?

    Ans: One-sixth of 36 is 6. Five-sixths is 5 times that or, 30 meters. 30 meters of cloth at $2 per meter will cost $60.

20. Warren had 12 shirts, and Oliver had 5 sixths as many less 4. How many shirts did Oliver have?

    Ans: Five-sixths of 12 shirts is 10 shirts. Oliver has now 10 - 4 = 6 shirts.

21. Dana had 60 special cards. She gave 2 sixths of them to her friend Barton, and 3 sixths to Benton. How many did she keep for herself? ***Ans:*** 10 cards.

22. If one meter of speaker wire costs 5 sixths of 36 cents, how many meters can you buy for 60 cents? ***Ans:*** 2 meters.

23. Two-thirds of $30 is $10 less than what Anil had. How much does Anil have? ***Ans:*** $30.

24. Fraser had 40 color pens. He gave 10 to Brown, and 2 sixths of the remaining to Seal. How many pens does he have left? ***Ans:*** 20 pens.

25. If a melon is divided into 7 equal parts, what are 1, 2, 3, 4, 5, and 6 of these parts called? ***Ans:*** One-seventh, 2 sevenths, 3 sevenths, 4 sevenths, 5 sevenths, 6 sevenths.

26. How many sevenths are there in a whole? ***Ans:*** 7.

27. If you give away one-seventh of something, how much do you have left? ***Ans:*** 6 sevenths.

28. If you give away 5 sevenths of something, how much do you have left? ***Ans:*** 2 sevenths.

29. What is one-seventh of 21? of 28? of 42? of 56? ***Ans:*** 3; 4; 6; 8.

30. What is 2 sevenths of 28? of 49? of 63? of 70? ***Ans:*** 8; 14; 18; 20.

    Note—First compute one-seventh of the number and then multiply it by 2.

31. What is 3 sevenths of 14? of 35? of 49? of 28? ***Ans:*** 6; 15; 21; 12.

32. What is 4 sevenths of 70? of 77? of 63? of 84? ***Ans:*** 40; 44; 36; 48.

33. What is 5 sevenths of 77? of 91? of 42? of 28? ***Ans:*** 55; 65; 30; 20.

34. What is 6 sevenths of 35? of 42? of 49? of 140? ***Ans:*** 30; 36; 42; 120.

35. Jim bought a bicycle and sold it for 6 sevenths of its cost. What portion of the cost did he lose? ***Ans:*** One-seventh.

36. Jim paid $70 for his bike, and sold it for 6 sevenths of its cost. How much money did he lose? ***Ans:*** $10.

37. Andy saw a bike for $210 and was told that the discount would be 2 sevenths of the cost. How much is the discount and what did Andy have to pay for the bike?

    Ans: One-seventh of $210 is $30. So the discount is $60 and Andy paid $150 for the bike.

38. Andy saw an another bike for $200 and was told that the discount on this bike would be one-fifth of the cost. How much is the discount and what would Andy have to pay for the bike?

    Ans: One-fifth of $200 is $40. So Andy would have to pay $160 for this bike.

39. What is the sum of one-sixth and one-seventh of 42? ***Ans:*** 13.

40. What is the sum of one-fifth and two-fifths of 20? ***Ans:*** 12.

41. What is the sum of one-third and one-sixth of 96? ***Ans:*** 48.

42. A student paid $70 for her math book, and 3 sevenths as much for her science book. After the term was over, she sold them both for $90. What was her loss?

    Ans: Three-sevenths of $70 is $30. $70 + $30 is $100, the whole cost. The loss is $100 - $90 = $10.

43. Three-sevenths of $56 is $6 more than what one book cost. What will 2 books cost at the same rate? ***Ans:*** $36.

44. Ben had $240, one-third of which he spent on a suit, one-fourth for a watch, and one-sixth for shoes. How much does he have left?

    Ans: One-third of 240 is 80, one-fourth is 60, and one-sixth is 40. So he spent 80 + 60 + 40 = 180 and has left 240 - 180 = $60.

45. A repair station has 40 barrels of oil. It used 3 fourths of it and then bought one-third as much as it sold. How many barrels of oil does the station have now?

    Ans: One-fourth of 40 is 10. The station used 3 fourths of 40, which is 3 times 10, or 30. Then it buys one-third of 30, which is 10. It had 40, used 30 and bought 10 more barrels. The total number of barrels it has now is 40 - 30 + 10 = 20 barrels.

46. Allen had $140. He gave 3 sevenths of it to his cousin, and spent 3 fourths of the rest on books. How much does he have left?

    Ans: He gave 3 sevenths of 140, or 60 to his cousin. He had left 140 - 60 = 80. He then spent 3 fourths of 80, or 60, and had left 80 - 60 = $20

47. Three-fourths of $40 is $20 more than what one book cost. What will 3 books cost at the same rate? ***Ans:*** $30.

48. Three-fifths of $50 is $20 more than what one book cost. What will 2 books cost at the same rate? ***Ans:*** $20.

49. Three-sevenths of $49 is $11 more than what one book cost. What will 2 books cost at the same rate? ***Ans:*** $20.

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Lesson 3 - Eighths and Ninths

1. IF anything is divided into 8 equal parts, what is one of these parts called? ***Ans:*** One-eighth.
2. What are 2, 3, 4, 5, 6 and 7 of these parts out of 8 called, and how many eighths are in a whole? ***Ans:*** 2 eighths, 3 eighths, 4 eighths, 5 eighths, 6 eighths, 7 eighths. Eight.
3. What is one-eighth of 24? 48? 72? 88? ***Ans:*** 3; 6; 9; 11.
4. What is 2 eighths of 32? 40? 56? 72? ***Ans:*** 8; 10; 14; 18.

   One-eighth of 32 is 4. So 2 eighths is 2 times 4 or 8.
5. What is 3 eighths of 16? 64? 80? 32? ***Ans:*** 6; 24; 30; 12.
6. What is 5 eighths of 8? 24? 48? 64? ***Ans:*** 5; 15; 30; 40.
7. One-eighth of 24 is how many times 3? ***Ans:*** One-eighth of 24 is 3 which is one times 3.
8. Three-eighths of 40 is how many times 5? ***Ans:*** One-eighth of 40 is 5 which is 3 times 5.
9. Four eighths of 80 is how many times 8? ***Ans:*** One-eighth of 80 is 10. 4 eighths is 4 times that, or 40, which is 5 times 8.
10. Five-eighths of 56 is how many times 7? ***Ans:*** One eighth of 56 is 7. 5 eighths is 5 times that, or 35, which is 5 times 7.
11. Six-eighths of 64 is how many times 12? ***Ans:*** One-eighth of 64 is 8. 6 eighths is 8 times that, or 48, which is 4 times 12.
12. Seven-eighths of 72 is how many times 3? ***Ans:*** 21.
13. Three-eighths of 32 is how many times one-third of 12?

    Ans: One-eighth of 32 is 4, and 3 eighths is 3 times 4, or 12. One-third of 12 is 4. The first part is 12, the second part of the problem is 4. 12 is 3 times 4.
14. Six-eighths of 40 is how many times one-fourth of 24?

    Ans: One-eighth of 40 is 5, and 6 eighths is 6 times 5, or 30; one-fourth of 24 is 6; 30 is 5 times that.
15. Four-eighths of 48 is how many times 2? ***Ans:*** 12.
16. One-eighth of 96 is how many times 6? ***Ans:*** 2.
17. Five-eighths of 56 is how many times 35? ***Ans:*** 1.
18. Two-thirds of 27 is how many times 2? ***Ans:*** 9.
19. Three-eighths of 48 is how many times 9? ***Ans:*** 2.
20. Seven-eighths of 72 is how many times 3? ***Ans:*** 21.
21. Five-eighths of 32 is how many times 5? ***Ans:*** 4.

22. Three-eighths of 48 is how many times 9? ***Ans:*** 2.

23. Two-eighths of 160 is how many times 8? ***Ans:*** 5.

24. Three-eighths of 320 is how many times 4? ***Ans:*** 30.

25. Two-thirds of 81 is how many times 9? ***Ans:*** 6.

26. Three-eighths of 16 is how many times 1? ***Ans:*** 6.

27. Five-eighths of 72 is how many times 5? ***Ans:*** 9.

28. Three-eighths of 64 is how many times 3? ***Ans:*** 8.

29. Two-thirds of 27 is how many times 3 fourths of 12? ***Ans:*** 2.

30. What is 2 ninths of 18? 27? 45? 36? ***Ans:*** 4; 6; 10; 8.

31. What is 3 ninths of 63? 72? 81? 27? ***Ans:*** 21; 24; 27; 9.

32. What is 4 ninths of 9? 36? 54? 81? ***Ans:*** 4; 16; 24; 36.

33. What is 5 ninths of 54? 72? 63? 27? ***Ans:*** 30; 40; 35; 15.

34. What is 6 ninths of 81? 18? 36? 90? ***Ans:*** 54; 12; 24; 60.

35. What is 7 ninths of 18? 99? 27? 108? ***Ans:*** 14; 77; 21; 84.

36. How much is three times 6 plus 2 thirds of 6?

    Ans: 3 times 6 is 18. One-third of 6 is 2, and 2 thirds of 6 is 2 times 2, or 4. 18 and 4 is 22.

37. How much is four times 12 plus 3 fourths of 12? ***Ans:*** 4 times 12 is 48; 3 fourths of 12 is 9; 48 + 9 = 57.

38. How much is 5 times 10 plus 3 fifths of 10? ***Ans:*** 56.

39. How much is 6 times 12 plus 3 sixths of 12? ***Ans:*** 78.

40. How much is 5 times 7 plus 4 sevenths of 7? ***Ans:*** 39.

41. How much is 9 times 8 plus 5 eighths of 8? ***Ans:*** 77.

42. 18 plus 7 ninths of 18 is how many? ***Ans:*** 32.

43. Two-ninths of 18 is how many times 2 thirds of 3?

    Ans: 2 ninths of 18 is 4; 2 thirds of 3 is 2; 4 is as many times 2 as 2 is contained in 4, or 2.

Contained in is a concept that means division. It is great for developing a comprehensive understanding of the process of division. We start with asking "what is contained in" and then move to direct division later.

44. Five-ninths of 27 is how many times 5 sixths of 6?

    Ans: 5 ninths of 27 is 15; 5 sixths of 6 is 5; 15 is as many times 5 as 5 is contained in 15, or 3.

45. Six-ninths of 54 is how many times 4 fifths of 15? ***Ans:*** 3.

46. Three-ninths of 72 is how many times 2 eighths of 16? ***Ans:*** 6.

47. Seven-eighths of 24 is how many times 7 eighths of 8? ***Ans:*** 3.

48. Lucy bought 60 items, and sold one-third of them to Bob, and 3 fifths of the remaining to Carl. How many itemss does she have now?

    Ans: Lucy sold Bob one-third of 60 items, or 20 items, and had 60 - 20, or 40 left. Three-fifths of 40 is 24. So she then has 40 - 24, or 16 items left.

    Which is bigger?

49. One-third of 24 or 2 thirds of 12? ***Ans:*** 8, Same.

50. One-third of 48 or 2 thirds of 24? ***Ans:*** 16, Same.

51. Two-thirds of 36 or one-third of 72? ***Ans:*** 24, Same.

52. One-third of 81 or 2 thirds of 57? ***Ans:*** 27 vs. 38, Second

53. One-third of 90 or 2 thirds of 60? ***Ans:*** 30 vs. 40, Second.

54. Two-thirds of 72 or one-third of 96? ***Ans:*** 48 vs. 32, First.

55. Two-fifths of 50 or 3 fifths of 40? ***Ans:*** 20 vs. 24, Second.

56. Two-fifths of 35 or 2 thirds of 18? ***Ans:*** 14 vs. 12, First.

57. Three-fifths of 60 or 2 thirds of 45? ***Ans:*** 36 vs. 30, First.

58. Four-fifths of 100 or one-third of 240? ***Ans:*** 80, Same.

59. Three-fifths of 80 or 2 thirds of 48? ***Ans:*** 16×3 vs. 16×2, First.

60. One-third of 72 or 2 fifths of 60? ***Ans:*** 24, Same.

61. Three-sevenths of 21 of 2 fifths of 40? ***Ans:*** 9 vs. 10, Second.

62. Three-sevenths of 42 or 2 less than 2 fifths of 50? ***Ans:*** 18, Same.

63. Four-sevenths of 63 or 20 more than 3 fifths of 200? ***Ans:*** 36 vs. 32, First.

64. Three-eighths of 56 or 8 less than 2 thirds of 72? ***Ans:*** 21 vs. 40, Second.

65. Three-sevenths of 70 or 10 less than 4 fifths of 50? ***Ans:*** 30, Same.

66. Four-fifths of 600 or 200 less than 3 fifths of 800? ***Ans:*** 480 vs. 280, First.

67. Three-eighths of 40 or 3 less than one-third of 57? ***Ans:*** 15 vs. 16, Second.

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The following pages contain the questions without the answers. It is much better if you read the questions to your student and wait for the answer. However some students may need to see or read the question. If this is the case, please print these sheets and hand them to the student.

Lesson one-Halves, Thirds, and Fourths

IF I divide an apple into two equal parts, what is one of these parts called? What are two of these parts called? ***Ans:*** One-half; a whole or one.

1. How many halves are in one apple?
2. What is one-half of 4? of 8? of 10? of 12?
3. What is one-half of 14? of 16? of 18? of 20?
4. What is one-half of 22? of 26? of 28? of 32?
5. If one pound of sugar costs 100 cents, what will one-half pound of sugar cost?
6. If a pound of coffee costs $10, then how many half pounds can you buy for $10?
7. If a pound of sugar costs $5, then how many half-pound boxes can you buy for $5?
8. Paul picked 18 apples and gave one-half to his brother. How many apples did he give to his brother?
9. Thompson bought 14 books, and sold one-half of them when the school term was over. How many of his books did he sell?
10. Phoebe had 40 peaches and gave one-half of them away. How many does she have left?
11. If I divide an apple into 3 equal parts, what is one of these parts called?
12. How many thirds are in one apple?
13. What are 2 parts and 3 parts of an apple cut into three pieces called individually?
14. How many one-thirds do you need to make a whole?
15. What is one-third of 6? of 9? of 12? of 15?
16. What is one-third of 21? of 24? of 30? of 36?
17. ames had $30 and spent one-third of it. How much did he spend?
18. Sanjay had 9 pears and Thomas had one-third as many as Sanjay. How many pears did Thomas have? ***Ans:*** Thomas had one-third of 9, which is 3 pears.
19. Lucy had 21 stamps and gave Mary one-third of them. How many stamps did she give to Mary?
20. The shop had 42 roses, and sold one-third of them to Bob. How many does he have left?
21. What is 2 thirds of 9? ***Ans:*** One-third of 9 is 3. If one-third of 9 is 3, then 2 thirds of 9 is 2 times 3, which is 6.

Note—In doing these problems, we will first find the unit fraction value and then multiply it with the numerator to get the answer. The unit fraction in problem 21 is (1/3). Its value is 3. In all sub-

sequent problems, always have student compute the unit fraction first. This is the only way to develop speed and understanding.

22. What is 2 thirds of 6? of 12? of 15? of 18? ***Ans:*** (3 x 2) = 4; (4 x 2) = 8; (5 x 2) = 10; (6 x 2) = 12. First find the unit fraction. One-third of 6 is 2. Two-thirds is twice that many, or 4.

23. What is 2 thirds of 24? of 30? of 27? of 33? ***Ans:*** 16; 20; 18; 22.

24. What is one-third of 123? What is two-thirds of 123?

25. What is one-third of 222? What is two-thirds of 222?

26. John had $21, and gave 2 thirds of his money to Sarah. How much did he give to Sarah?

27. There were 27 cherries in the bowl. You ate 2 thirds of them. How many did you eat?

28. If you give away one-third of something, how many thirds do you have left?

29. 29. Rudy had a rock collection of 33 rocks. He gave his sister for her birthday 2 thirds of it. How many did he keep?

30. Daniel had $36 and lost 2 thirds of it somewhere. How much money does he have left?

31. If I divide an apple into 4 equal parts, what are 1, 2, and 3, of these parts called?

32. How many fourths of an apple are in a whole apple?

33. What is one-fourth of 4? of 8? of 20? of 32?

34. What is one-fourth of 12? of 24? of 16? of 48?

35. What is 2 fourths of 24? of 16?; of 28? of 36?

36. If one-fourth of 20 is 5, then what is 2 fourths?

37. What is 3 fourths of 20? of 24? of 12? of 16?

38. What is 3 fourths of 120?

39. The ski shop had 44 snow boards. The shop rented 2 fourths of them on Friday. How many snow boards are left in the shop?

40. If a meter of cloth costs $8, then what will 3 fourths of a meter cost?

41. If I give away one-third of some thing, how many thirds do I have left?

42. What is one-half and one-third of 24?

43. Jessica had 24 gum-balls. She gave half of them to Margie and a third of them to Cathy. How many gum-balls does she have left? ***Ans:*** One-half of 24 is 12. One-third of 24 is 8. She gave 12 gum-balls to Margie and 8 balls to Cathy

44. Harley is 24 years old and Townsend is 8 fourths as old. How old is Townsend?

45. What is the sum of one half and one-third of 36?

46. A farmer harvested 36 melons. A shop bought half of them, and one-third were too green to be sold. How many ripe melons does the farmer have left? .

47. A store owner has 40 boxes of sneakers. He sold 3 fourths of them. How many boxes does he have left? He then bought one-third as much as he sold. How many boxes of sneakers does the store have now?

48. A store owner has 20 jackets. She sold 2 fourths of them. How many does she have left? She then bought one-half as much as she sold. How many are there now?

49. What is an another way of saying one-fourth?

50. Why is the coin “quarter” called a quarter?

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Lesson 2 - Fifth, Sixth, and Seventh

IF you divide an orange into 5 equal parts, what are 1, 2, 3, and 4 of these parts called?

Ans: One-fifth, 2 fifths, 3 fifths. 4 fifths.

1. How many fifths are in one orange?
2. What is one-fifth of 10? of 25? of 15? of 30?
3. What is 2 fifths of 15? of 30? of 45? of 20?
   Note—First compute one-fifth and then multiply it by 2.
4. What is 3 fifths of 10? of 30? of 25? of 55?
   Note—First compute one-fifth and then multiply it by 3.
5. What is 4 fifths of 55? of 35? of 40? of 50?
   Note—First compute one-fifth and then multiply it by 4.
6. Mary has 15 blue shells, and Rachel has 2 fifths as many. How many blue beads does Rachel have?
7. Susan is 25 years old, and her sister is 4 fifths as old. How old is her sister?
8. Roland is 35 years old, and his sister is 4 fifths as old. How old is his sister?
9. A horse cost $1000, and a saddle cost 3 fifths as much as the horse. What is the cost of the saddle?
10. Mary had 40 lambs and she lost 20 of them. After a lot of searching, she found 3 fifths of her lambs. How many lambs does she have now?
    A dairyman owned 50 cows. He sold 4 fifths of them, and then bought 32 new cows. How many does he have now?
11. If you cut a melon into 6 equal parts, what are these parts called?
12. How many sixths are there in a single thing? ***A***
13. What is 2 sixths of 24? of 18? of 36? of 60?
    Note—First compute one-sixth of the number and then multiply it by 2.
14. What is 3 sixths of 12? of 42? of 30? of 66?
15. What is 4 sixths of 6? of 36? of 48? of 54?
16. What is 5 sixths of 18? of 54? of 24? of 72?
17. Rob went to a festival and brought back 48 flags. He gave 3 sixths of the flags to Megan and 2 sixths to Morgan. How many flags did he give to both?

18. What will 5 sixths of 36 meters of cloth cost at the rate of $2 a meter?

19. Warren had 12 shirts, and Oliver had 5 sixths as many less 4. How many shirts did Oliver have?

20. Dana had 60 special cards. She gave 2 sixths of them to her friend Barton, and 3 sixths to Benton. How many did she keep for herself?

21. If one meter of speaker wire costs 5 sixths of 36 cents, how many meters can you buy for 60 cents?

22. Two-thirds of $30 is $10 less than what Anil had. How much does Anil have? ***Ans:*** $30.

23. Fraser had 40 color pens. He gave 10 to Brown, and 2 sixths of the remaining to Seal. How many pens does he have left?

24. If a melon is divided into 7 equal parts, what are 1, 2, 3, 4, 5, and 6 of these parts called?

25. How many sevenths are there in a whole?

26. If you give away one-seventh of something, how much do you have left?

27. If you give away 5 sevenths of something, how much do you have left?

28. What is one-seventh of 21? of 28? of 42? of 56?

29. What is 2 sevenths of 28? of 49? of 63? of 70?
    Note—First compute one-seventh of the number and then multiply it by 2.

30. What is 3 sevenths of 14? of 35? of 49? of 28?

31. What is 4 sevenths of 70? of 77? of 63? of 84?

32. What is 5 sevenths of 77? of 91? of 42? of 28?

33. What is 6 sevenths of 35? of 42? of 49? of 140?

34. Jim bought a bicycle and sold it for 6 sevenths of its cost. What portion of the cost did he lose?

35. Jim paid $70 for his bike, and sold it for 6 sevenths of its cost. How much money did he lose?

36. Andy saw a bike for $210 and was told that the discount would be 2 sevenths of the cost. How much is the discount and what did Andy have to pay for the bike?

37. Andy saw an another bike for $200 and was told that the discount on this bike would be one-fifth of the cost. How much is the discount and what would Andy have to pay for the bike?

38. What is the sum of one-sixth and one-seventh of 42?

39. What is the sum of one-fifth and two-fifths of 20?

40. What is the sum of one-third and one-sixth of 96?

41. A student paid $70 for her math book, and 3 sevenths as much for her science book. After the term was over, she sold them both for $90. What was her loss?

42. Three-sevenths of $56 is $6 more than what one book cost. What will 2 books cost at the same rate?

43. Ben had $240, one-third of which he spent on a suit, one-fourth for a watch, and one-sixth for shoes. How much does he have left?

44. A repair station has 40 barrels of oil. It used 3 fourths of it and then bought one-third as much as it sold. How many barrels of oil does the station have now?

45. Allen had $140. He gave 3 sevenths of it to his cousin, and spent 3 fourths of the rest on books. How much does he have left?

46. Three-fourths of $40 is $20 more than what one book cost. What will 3 books cost at the same rate?

47. Three-fifths of $50 is $20 more than what one book cost. What will 2 books cost at the same rate?

48. Three-sevenths of $49 is $11 more than what one book cost. What will 2 books cost at the same rate?

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Lesson 3 - Eighths and Ninths

1. IF anything is divided into 8 equal parts, what is one of these parts called? ***Ans:*** One-eighth.
2. What are 2, 3, 4, 5, 6 and 7 of these parts out of 8 called, and how many eighths are in a whole?
3. What is one-eighth of 24? 48? 72? 88?
4. What is 2 eighths of 32? 40? 56? 72?
   One-eighth of 32 is 4. So 2 eighths is 2 times 4 or 8.
5. What is 3 eighths of 16? 64? 80? 32?
6. What is 5 eighths of 8? 24? 48? 64?
7. One-eighth of 24 is how many times 3?
8. Three-eighths of 40 is how many times 5?
9. Four eighths of 80 is how many times 8?
10. Five-eighths of 56 is how many times 7?
11. Six-eighths of 64 is how many times 12?
12. Seven-eighths of 72 is how many times 3?
13. Three-eighths of 32 is how many times one-third of 12?
14. Six-eighths of 40 is how many times one-fourth of 24?
15. Four-eighths of 48 is how many times 2?
16. One-eighth of 96 is how many times 6?
17. Five-eighths of 56 is how many times 35?
18. Two-thirds of 27 is how many times 2?
19. Three-eighths of 48 is how many times 9?
20. Seven-eighths of 72 is how many times 3?
21. Five-eighths of 32 is how many times 5?
22. Three-eighths of 48 is how many times 9?
23. Two-eighths of 160 is how many times 8?
24. Three-eighths of 320 is how many times 4?
25. Two-thirds of 81 is how many times 9?

26. Three-eighths of 16 is how many times 1?
27. Five-eighths of 72 is how many times 5?
28. Three-eighths of 64 is how many times 3?
29. Two-thirds of 27 is how many times 3 fourths of 12?
30. What is 2 ninths of 18? 27? 45? 36?
31. What is 3 ninths of 63? 72? 81? 27?
32. What is 4 ninths of 9? 36? 54? 81?
33. What is 5 ninths of 54? 72? 63? 27?
34. What is 6 ninths of 81? 18? 36? 90?
35. What is 7 ninths of 18? 99? 27? 108?
36. How much is three times 6 plus 2 thirds of 6?
37. How much is four times 12 plus 3 fourths of 12?
38. How much is 5 times 10 plus 3 fifths of 10?
39. How much is 6 times 12 plus 3 sixths of 12?
40. How much is 5 times 7 plus 4 sevenths of 7?
41. How much is 9 times 8 plus 5 eighths of 8?
42. 18 plus 7 ninths of 18 is how many?
43. Two-ninths of 18 is how many times 2 thirds of 3?

Contained in is a concept that means division. It is great for developing a comprehensive understanding of the process of division. We start with asking "what is contained in" and then move to direct division later.

44. Five-ninths of 27 is how many times 5 sixths of 6?
45. Six-ninths of 54 is how many times 4 fifths of 15?
46. Three-ninths of 72 is how many times 2 eighths of 16?
47. Seven-eighths of 24 is how many times 7 eighths of 8?
48. Lucy bought 60 items, and sold one-third of them to Bob, and 3 fifths of the remaining to Carl. How many itemss does she have now?

    Which is bigger?
49. One-third of 24 or 2 thirds of 12?
50. One-third of 48 or 2 thirds of 24?

51. Two-thirds of 36 or one-third of 72?
52. One-third of 81 or 2 thirds of 57?
53. One-third of 90 or 2 thirds of 60?
54. Two-thirds of 72 or one-third of 96?
55. Two-fifths of 50 or 3 fifths of 40?
56. Two-fifths of 35 or 2 thirds of 18?
57. Three-fifths of 60 or 2 thirds of 45?
58. Four-fifths of 100 or one-third of 240?
59. Three-fifths of 80 or 2 thirds of 48?
60. One-third of 72 or 2 fifths of 60?
61. Three-sevenths of 21 of 2 fifths of 40?
62. Three-sevenths of 42 or 2 less than 2 fifths of 50?
63. Four-sevenths of 63 or 20 more than 3 fifths of 200?
64. Three-eighths of 56 or 8 less than 2 thirds of 72?
65. Three-sevenths of 70 or 10 less than 4 fifths of 50?
66. Four-fifths of 600 or 200 less than 3 fifths of 800?
67. Three-eighths of 40 or 3 less than one-third of 57?

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Done to here.